Absence of Nonlocal Counter-terms in the Gauge Boson Propagator in Axial -type Gauges

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Abstract

We study the two-point function for the gauge boson in the axial-type gauges.We use the exact treatment of the axial gauges recently proposed that is intrinsically compatible with the Lorentz type gauges in the path-integral formulation and has been arrived at from this connection and which is a "one-vector" treatment. We find that in this treatment, we can evaluate the two-point functions without imposing any additional interpretation on the axial gauge $1/(\eta.\mathrm{q})^q$ type poles.The calculations are as easy as the other treatments based on other known prescriptions.Unlike the "uniform-prescription" /L-M prescription, we note,here, the absence of any non-local divergences in the 2-point proper vertex. We correlate our calculation with that for the Cauchy Principal Value prescription and find from this comparison that the 2-point proper vertex differs from the CPV calculation

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only by finite terms.For simplicity of treatment, the divergences have been calculated here with $\eta^2 > 0$ and these have a smooth light cone limit.

1 Introduction

The calculations in gauge theories have been done primarily using the Lorentz-type [including the R_ξ-gauges] and the axial type gauges [1]. The Lorentz type gauges have the several desirable properties of Lorentz- covariance ,ease of calculations and the availability of a gauge parameter to verify the gauge-independence. They , however, are burdened with having to include the diagrams involved with the Faddeev-Popov ghosts.As a result of this, another set of gauges ,the axial-type gauges[these include the light-cone [LCG] and the temporal gauges], have also been employed in the Standard Model [SM] calculations.These gauges purport to have decoupling of ghosts [2,3,4] and consequently require a fewer set of diagrams.The disadvantages these gauges suffer from are the lack of manifest covariance, extra counter-terms arising from an additional vector η { or *two* additional vectors in case of the Leibbrandt-Mandelstam/Uniform prescriptions [2,3]} and the problem of how to correctly deal with the spurious singularities of the form $1/(\eta.\mathrm{q})^q$.Doubts have also been expressed [5] about the exceptional advantages of the axial-type gauges;nonetheless they have been found useful in practice. The crucial problem of $1/(\eta.\mathrm{q})^q$ -type singularities has been widely addressed over several decades [2,3,6-8]. The early use of principal value prescription came under cloud on account of several difficulties encountered with it. It was found that the PVP could not be used for the light-cone gauges for several reasons [3,4].Further, it was shown that the PVP does not give the correct behavior for the Wilson loop to O[g^4] compatible with the calculation in Feynman gauge[9]. To overcome these difficulties, the Leibbrandt-Mandelstam prescription [2,3,4] was proposed and applied to the light-cone gauges.It was further generalized to the other axial gauges through what is called the uniform prescription [2,3]. One of the drawbacks of the L-M/Uniform prescriptions has been the presence of the non-local counterterms in proper vertices to all orders. Another drawback is, of course,that the presence of two vectors leads to a larger set of possible divergences. Bassetto and coworkers have developed the techniques for dealing with non-local divergences [2,3]. Sev-

eral other ways of dealing with the $1/(\eta.\mathrm{q})^q$-type singularities have also been proposed [6-8].

A new way of attacking the problem of $1/(\eta.\mathrm{q})^q$ -type singularities in axial type-gauges was formulated recently.This was based on establishing the relation between the Green's functions in the Lorentz and the axial-type gauges in the path-integral formulation.This relation was, in turn, based on what was called the finite field-dependent BRS [FFBRS] transformation [10]. According to this view-point, the crucial question is how to develop an *intrinsically well-defined* path-integral treatment in any other gauge that is compatible to that in the Lorentz gauges *by construction* that, we understand have no analogous difficulties. Such a well-defined treatment for defining the path-integral in axial-type gauges [and which is also applicable to host of other gauges [11]]has been established using the FFBRS transformation [12,13,7,8].It is expected that this path-integral so constructed should provide answers to all the questions regarding the various difficulties associated with the variety of the non-covariant gauges [e.g. axial, light-cone, temporal [2,3], Coulomb [14] that are thought to arise from the ill-definedness of the naive treatment. In other words,as we have emphasized, we expect that the way to deal with these difficulties does not require an ad hoc augmentation of rules as to how the diagrams are calculated, but is already contained in the process outlined earlier in references [11,12,13].The above expectation ,based on rigorous formal arguments, derives concrete support from several works [7,8,15]. For example following the above outlook [16], we have established an effective treatment of the axial gauge propagator [7,8].We have also shown that the Wilson loop for axial and Lorentz gauges has the same value to $\mathrm{O}[\mathrm{g}^4]$ for a wide class of loops [15].

With this in mind ,we perform a simple one-loop calculation of the 2-point proper vertex in pure gauge theories.We have several motivations to perform this simple calculation. According to the view-point mentioned earlier, our calculation does not have any arbitrariness associated with it and is in a formalism intrinsically compatible with the Lorentz gauges.Our calculation involves evaluations of terms having contour integrals and those with effective δ-function terms. We exhibit that our calculations are no more cumbersome than those with other prescriptions and that the simplifications that are normally associated with dimensional regularization [DR] continue to hold in our treatment. We find it easy and convenient to correlate our calculations

with those with PVP [1] and point out the additional contributions. We show how these can be dealt with and show that these are in fact finite. Our method ,*without any interpretation given to Feynman integrals,* allows one to verify that the ghost diagram does not contribute to the *proper* two point function.

We summarize the plan of the paper.In section 2, we introduce the notations and the past results obtained from the FFBRS treatment [11,12,7,8]. In section 3, we give the procedure for evaluation of the three diagrams, both the contour integral-type contribution and the effective term-type contributions.We show that the evaluations are in no way more involved as compared to those with say, CPV or L-M prescriptions. We emphasize that we, in no way, need to *interpret* any of the term in the Feynman diagrams. We show here ,in particular, that the results that we normally expect for the tadpole diagram in the dimensional regularization and from the ghost-decoupling for the ghost diagram hold in this treatment also. Again, the ghost diagram does not contribute *without* requiring any assumptions made about $1/\eta.q$ type terms.In section 4, we summarize our results and indicate heuristically the direction one can take to tackle the question of locality of divergences in higher orders and for other n-point functions. Appendix B deals with the ghost diagram.

2 Preliminaries

We work in the axial-type gauges with a gauge parameter λ and

$$S^A_{\text{eff}} = -\frac{1}{2\lambda}\int d^4x(\eta\cdot A)^2 \tag{1}$$

and regard $\eta\cdot A = 0$ as the $\lambda \to 0$ limit of the above family of gauges. We also have the ghost action:

$$S^A_{\text{gh}} = -\int d^4x\bar{c}\eta^\mu D_\mu c. \tag{2}$$

In references]7] and]8], we derive the treatment of the axial gauge poles by connecting the axial gauges to the Lorentz gauges established in earlier

[1] We do not necessarily imply correctness or otherwise of the use of PVP by this comparison.

works]12] and]14].. We found the exact axial gauge propagator in this way and a much simpler effective treatment for it. The propagator reads

$$\tilde{G}^{0A}_{\mu\nu} = \tilde{G}^{0L}_{\mu\nu} \ + \left[\Big(k_\mu k_\nu \Sigma_1 + \eta_\mu k_\nu \Sigma_2 \Big) ln\Sigma_3 + (k \to -k; \mu \leftrightarrow \nu) \right] \tag{3}$$

where

$$\Sigma_1 \equiv \frac{-(k^2 - i\eta\cdot k)\Big(\frac{\eta\cdot k + i\eta^2}{k^2 - i\eta\cdot k} + i\lambda - \frac{(1-\lambda)\eta\cdot k}{k^2+i\epsilon}\Big)}{\epsilon\Sigma}$$

$$\Sigma_2 \equiv \frac{-(k^2 - i\eta\cdot k)\Big(-\Big[\frac{k^2 + i\eta\cdot k}{k^2 - i\eta\cdot k}\Big] + 1 - \frac{i\epsilon(1-\lambda)}{k^2+i\epsilon}\Big)}{\epsilon\Sigma}$$

$$\Sigma_3 \equiv \frac{-i(\eta\cdot k + \epsilon)(k^2 + i\epsilon\lambda)}{(k^2 + i\epsilon)\Big(-i\epsilon\lambda - \sqrt{k^4 - (k^2 + i\epsilon\lambda)\Big[k^2 + \frac{(\eta\cdot k)^2 + i\epsilon\eta^2}{k^2+i\epsilon}\Big]}\Big)},$$

and

$$\Sigma \equiv \Big[(1-\lambda)[(\eta\cdot k)^2 + 2ik^2\eta\cdot k] + i\epsilon k^2(1-2\lambda) + \lambda(k^2+i\epsilon)^2 + \eta^2(k^2+i\epsilon)\Big]. \tag{4}$$

Despite the formidable appearance of the above, a much simpler effective treatment was also established in [7[. It was shown that coordinate space propagator $D_{\mu\nu}(x,y)$ should be evaluated as shown below. We shall take $\eta_0 \neq 0$ and in fact let $\eta_0 = 1$. [2]Below, C represents a contour from $(-\infty,\infty)$ along the real k_0-axis except a semicircular dip of radius $\alpha\sqrt{\epsilon}(\alpha >> 1)$ below $\eta\cdot k = 0$. We have

$$D_{\mu\nu}(x,y) = \int d^3k \int_C dk^0 e^{-ik\cdot(x-y)} D^0_{\mu\nu}(k) + \int d^3k \int_{-\infty}^{\infty} e^{-ik\cdot(x-y)} D^{\rm extra}_{\mu\nu}(k), \tag{5}$$

with D^0 as the usual axial propagator away from $\eta\cdot k = 0$ axis, viz

$$D^0_{\mu\nu}(k)_{=} - \frac{1}{k^2+i\epsilon}(g_{\mu\nu} - \frac{k_\mu\eta_\nu + k_\nu\eta_\mu}{\eta\cdot k} + k_\mu k_\nu \frac{\lambda k^2 + \eta^2}{(\eta\cdot k)^2}\Big), \tag{6}$$

[2]We shall find it convenient to deal with the case $\eta^2 > 0$ as this makes the treatment simpler.

and [7[

$$D_{\mu\nu}^{\rm extra}(k) = \delta\Big(k^0 - \frac{1}{2}\sqrt{\frac{\epsilon\eta^2}{i}} - \vec{\eta}\cdot\vec{k}\Big)\Big(k_\mu k_\nu\Big[-i\sqrt{\frac{i\eta^2}{\epsilon}}a_{1(0)} - \frac{i\eta^2}{2}a_{1(1)}\Big]$$
$$+\eta_\mu k_\nu\Big[-\frac{i\eta^2}{2}a_{2(1)}\Big] + \eta_\nu k_\mu\Big[-\frac{i\eta^2}{2}a_{3(1)}\Big]\Big)$$
$$= \delta\Big(k^0 - \frac{1}{2}\sqrt{\frac{\epsilon\eta^2}{i}} - \vec{\eta}\cdot\vec{k}\Big)$$
$$\times\Big(-\frac{2\pi\eta^2}{(\eta^2+i\epsilon)[(\vec{\eta}\cdot\vec{k})^2-\vec{k}^2]}\Big[i\sqrt{\frac{i\eta^2}{\epsilon}} + \frac{\eta^2}{\eta^2+i\epsilon}\Big]k_\mu k_\nu - \frac{2\pi}{(\eta^2+i\epsilon)[(\vec{\eta}\cdot\vec{k})^2-\vec{k}^2]}(\eta_\mu k_\nu + k_\nu\eta_\mu)\Big). \tag{7}$$

We [3] shall find it beneficial to compare our calculations with the Cauchy Principle Value prescription (CPV) calculations for which

$$\frac{1}{(\eta\cdot k)^\beta} \rightarrow \frac{1}{2}\lim_{\mu\rightarrow 0}\Big[\frac{1}{(\eta\cdot k + i\mu)^\beta} + \frac{1}{(\eta\cdot k - i\mu)^\beta}\Big]. \tag{8}$$

3 One-Loop Calculations

In this section, we shall pursue the one-loop calculations with the present formalism. We illustrate how treatment of the Feynman integrals involving contours of the type as in (5), and that of the extra effective terms. We, in particular, emphasize that our calculations are no way more laborious, despite its new appearance, than those performed with prescriptions such as CPV and Uniform prescriptions]2],]3]. We further wish to emphasize that our calculations are done from first principles and are compatible with those of Lorentz-type gauges]13],]11].by construction and involve no arbitrariness like the other prescriptions . Our method, in addition, has only one vector η unlike the uniform prescription and thus does not increase the number of possible counterterms.. Moreover, we find, as shown below, that we do not encounter non-local divergence (in this simple example) found with uniform/ML prescription and we suspect this to hold in higher order calculations. Unlike CPV, this treatment has been shown explicitly to preserve a

[3]We note that for $\eta^2 > 0$, the denominator $(\vec{\eta}\cdot\vec{k})^2 - \vec{k}^2$ never vanishes and we can simplify by dropping ϵ that went along with it.

large class of Wilson lops (i.e. leads to the same value for it as in Lorentz gauges]15]). We find that the divergences found have a smooth limit $\eta^2 \to 0$ and leads to no nonlocal divergences.

We shall find it convenient to correlate our calculations with those done with CPV prescription. We shall in fact show that the contributions, over and above CPV, both from the contour integral and the effective terms are finite. We shall successively deal with the three diagrams in Fig 1. We shall show that in our effective treatment also, the tadpole diagram as well as the ghost diagrams vanish exactly (i.e. without an interpretation given to the propagator near $\eta \cdot k = 0$).

As for the tadpole diagram, it is proportional to

$$\int d^4k D_\mu^\mu(k) = \int d^3k \int_C dk_0 D_\mu^{0\ \mu}(k) + \int d^3k \int dk_0 D_\mu^{\text{extra}\ \mu}(k). \tag{9}$$

From (6),

$$\begin{aligned} D_\mu^{0\mu}(k) &= -\frac{2}{k^2+i\epsilon} - \frac{\lambda k^2+\eta^2}{(\eta\cdot k)^2} + O(\epsilon) \\ &= -\frac{2}{k^2+i\epsilon} - \lambda - \lambda\eta \ \ \cdot k - \frac{\lambda[(\vec{\eta}\cdot\vec{k})^2 - \vec{k}^2] + \eta^2}{(\eta\cdot k)^2}. \end{aligned} \tag{10}$$

We note that the term $(-\lambda)$ can be dropped ($\int d^nk = 0$). The contour integral over C can be obtained by closing it below for the first and the last terms; the last term then does not contribute. We obtain, using symmetric integration for the third term:

$$\int_C D_\mu^{0\mu}(k) = \frac{2\pi i}{|\vec{k}|} - \lambda\pi i\vec{\eta}\cdot\vec{k} \tag{11}$$

We, the, note that the first term in (9) vanishes noting that in dimensional regularisation,

$$\int \frac{d^{n-1}k}{|\vec{k}|} = \int d^{n-1}k\vec{\eta}\cdot\vec{k} = 0. \tag{12}$$

We note that the second term in (9) vanishes noting from (7) that

$$D_\mu^{\text{extra}\mu}(k) = \text{a } \vec{\mathrm{k}} - \text{independent constant}, \tag{13}$$

and $\int d^{n-1}k = 0$. Thus, the diagram vanishes here also.

We shall, next, take up diagram 1(b). We shall first establish the treatment for the graph in terms of a contribution over C and an effective contribution. Up to overall factors, the graphs is of the form,

$$\int d^4k V_{\mu\rho\lambda}(p,k,-k-p)V_{\nu\kappa\sigma}(-p,k+p,-k)D^{\sigma\rho}(k)D^{\lambda\kappa}(k+p), \qquad (14)$$

where $D^{\sigma\rho}$ is the exact propagator found in [7], [8]. We shall reduce the expression (14) in terms of the contour integral and an effective term. We do this in Appendix A for $\eta\cdot p\neq 0$. As shown in Appendix A, and not surprisingly, we find:

$$\int_{-\infty}^{\infty} dk_0 VVDD = \int_{C_0} dk_0 VVD^0(k)D^0(k+p) + \int dk_0 VVD^0(k+p)D^{\text{extra}}(k) + \int dk_0 VVD^0(k)D^{\text{extra}}(k+p). \qquad (15)$$

Here C_0 is the contour running along the real k_0-axis except for two semicircular dips of radius $\alpha\sqrt{\epsilon} << |\eta\cdot p|$ centered at $\eta\cdot k=0$ and $\eta\cdot(k+p)=0$.

In Appendix A, we have further established a simplification in the first term on the right hand side of (15). It is shown there that in evaluating the term, we only have to consider integrals of the form:

$$\int_C d^n k \frac{P(k,p)}{(k^2+i\epsilon)[(k+p^2+i\epsilon]\zeta^\alpha}, \; 0\leq\alpha\leq 2, \qquad (16)$$

where C is the contour with only a single semicircular dip around $\eta\cdot k=0$, and $D^0(k)$ is the naive propagator of (6).

We show that the divergences in the integral of the form (16) can in fact, be related to their evaluations in the CPV scheme. We recall the result for

$$\int_{\text{CPV}} d^n k \frac{P(k,p)}{(k^2+i\epsilon)[(k+p^2+i\epsilon]\zeta^\alpha}$$
$$= \lim_{\mu\to 0}\Big(\frac{1}{2}\int d^n k\int_{-\infty}^{\infty} dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon](\zeta-i\mu)^\alpha}$$
$$+\int d^n k\int_{-\infty}^{\infty} dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon](\zeta+i\mu)^\alpha}\Big). \qquad (17)$$

In the first integral on the right hand side of (17), we can deform the contour along the real k_0-axis to C (i.e. with a dip at $\zeta=0$) and allow $\mu\to$. Similarly,

in the second integral, we can deform the contour to C' (with a bump above at $\zeta = 0$) and allow $\mu \to$. Thus, we have

$$\int_{\text{CPV}} dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p^2+i\epsilon]\zeta^\alpha} = \frac{1}{2}\Big(\int_C + \int_{C'}\Big) dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p^2+i\epsilon]\zeta^\alpha}. \tag{18}$$

We thus see that

$$\int_C dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p^2+i\epsilon]\zeta^\alpha} - \int_{\text{CPV}} k_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon]\zeta^\alpha}$$
$$= \frac{1}{2}\oint \frac{P(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon]\zeta^\alpha} \tag{19}$$

where $\oint_C$ goes over a circular contour of radius $= \alpha\sqrt{\epsilon}$ around $\zeta = 0$.

In evaluating the diagram 1(b), we then have to evaluate integrals of the form

$$I_1 = \int d^3k \int_{C_0} dk_0 V_{\mu\rho\lambda} V_{\nu\xi\sigma} D^{0\sigma\rho}(k) D^{0\lambda\xi}(k+p) \tag{20}$$

and

$$I_2 = \int d^3k \int_{C_0} dk_0 V_{\mu\rho\lambda} V_{\nu\xi\sigma} D^{0\sigma\rho}(k) D^{\text{extra}\lambda\xi}(k+p) \tag{21}$$

(and an analogous term I'_2 with $D^{\text{extra}\sigma\rho}(k)D^{0\lambda\xi}(k+p)$). In (20), we keep aside the Feynman gauge term, as this has only a local divergence. in other terms in (20), we apply the simplifications due to (i) tree WT-identity for $p_\mu V^\mu$ etc. and (ii) partial fraction indicated in Appendix A (See equation (A.5)) and arrive at integrals of the form of (19). These can be evaluated firstly by the residue theorem and then by dimensional regularization. Integrals involved in (21) are analogously evaluated by first doing the k_0 integration and then evaluating the $d^{n-1}k$ integral in dimensional regularization. We have verified that the integrals involved (given below) in either case are finite in dimensional regularization. These integrals are[4]

$$\int d^dk \frac{(1, k_\mu, k_\mu k_\nu)\zeta^{0,1}}{[p+k)^2+i\epsilon](\vec{k}^2 - i\epsilon)(\zeta+\zeta_0)^{0,1,2}} \delta(\zeta - \frac{1}{2}\sqrt{-\epsilon\eta^2 i})$$

[4]The integrals above generate several (d-1)-dimensional integrals regularized dimensionally.Such odd-dimensional integrals occur in statistical field theory in 3-dimensions and have been extensively, for example , in Itzykson and Drouffe "Statistical field theory" Vol.I.;Cambridge University Press, Cambridge, 1989.

$$\int d^d k \frac{(1, k_\mu, k_\mu k_\nu)\zeta^{0,1}}{(k^2 - i\epsilon)(\zeta + \zeta_0)^{0,1,2}} \delta(\zeta - \frac{1}{2}\sqrt{-\epsilon\eta^2 i}) \tag{22}$$

and those differing from the above by terms of $O(\sqrt{\epsilon})$ in various factors in the denominator. We further need the following integrals:

$$\begin{aligned} &\int d^4 k \frac{\partial}{\partial \zeta}\Big[\frac{k_\mu k_\nu}{k^2 (k+p)^2}\Big]\delta(\zeta); \ \int d^4 k \frac{\partial}{\partial \zeta}\Big[\frac{k_\mu}{k^2}\Big]\delta(\zeta); \\ &\int d^4 k \frac{\partial}{\partial \zeta}\Big[\frac{\eta \cdot (k+p)(k+p)_\mu k_\nu}{k^2 (k+p)^2}\Big]\delta(\zeta) \end{aligned} \tag{23}$$

(We also need integrals related to the above by a shift of variable). We evaluate these integrals for p_0 purely imaginary. We assume the analytic continuation of the divergent part (which in our case is local) to real p_0 values.

Finally, we deal with the ghost loop diagram in the Appendix B and show that it vanishes.

We, thus,see that the divergences in the gluon two point proper vertex in the one loop are identical to those with CPV prescription.

4 Conclusions and Comments

In this section, we summarize the results we have obtained. We started by emphasizing that it is not required to interpose an interpretation of the $1/(\eta \cdot q)$ type of poles, and that the correct treatment of the axial gauge propagator is obtained by its comparison with the Lorentz gauge path integral through FFBRS transformations. It involves a well-defined contour integral and an effective δ-function term.The 2-point calculation presented here is a first direct application of the propagator. It illustrates and brings out several points:(i) The calculation, which has no arbitrariness about it, is in no way more involved than other prescriptions; (ii) It has only one vector η already present in the axial gauges required; thus limiting the number of the possible counterterms in general; (iii) The usual expectations about the tadpole diagram in DR and the ghost diagram are still valid in this formulation .With

the calculations so performed, we find, unlike the L-M /Uniform prescription, absence of any nonlocal counterterms in the 2-point proper vertex.

It is possible that like the CPV prescription [2] that has one vector needed, this treatment, a one-vector treatment itself, may also have only local counterterms necessary for other n-point proper vertices and higher loop orders.

We, finally,comment on the possible approach to the study of locality of counter terms.It is based on the result that correlates the axial Green's functions to the Lorentz Green's functions that was obtained in references 13 and 7.The result [see e.g.(46) of Ref.7] is valid for arbitrary Green's functions [as well as operator Green's functions] .To illustrate the point, we state the result for the 2-point function for which it reads:

$$iG^{A\ \alpha\beta}_{\mu\nu}(x-y) = iG^{L\ \alpha\beta}_{\mu\nu}(x-y) + i\int_0^1 d\kappa \int \mathcal{D}\phi e^{iS^M_{\rm eff}[\phi,\kappa]-i\epsilon\int(A^2/2-\bar{c}c)d^4x}$$
$$\times\Big((\mathrm{D}_\mu c)^\alpha(x)A^\beta_\nu(y) + A^\alpha_\mu(x)(\mathrm{D}_\nu c)^\beta(y)\Big)\int d^4z\bar{c}(z)(\partial\cdot A^\gamma - \eta\cdot A^\gamma)(z) \tag{24}$$

The above relation gives the value of the exact axial propagator *compatible to the Green's functions in Lorentz gauges* . The result is exact to all orders.As mentioned in Ref.13, to any finite order in g, the right hand side can be evaluated by a finite sum of Feynman diagrams. These diagrams involve ,in particular, ordinary propagators obtained from the mixed gauge effective action and a final κ-integral. A study of the form of the contributions to the last term above should enable one to determine about the locality of counterterms. This will be left for another work [17].

Appendix A

We shall take up diagram 1(b). We shall establish the treatment for the graph in terms of a contribution over C and an effective contribution. Up to overall factors, the graphs is of the form

$$\int d^4kV_{\mu\rho\lambda}(p,k,-k-p)V_{\nu\kappa\sigma}(-p,k+p,-k)D^{\sigma\rho}(k)D^{\lambda\kappa}(k+p), \tag{A1}$$

where $D_{\sigma\rho}(k)$ is the exact propagator found in]7,8]. We shall reduce the expression (14) in terms of the contour integral and and an effective term.

We do this for $\eta \cdot p \neq 0$. We chose ϵ such that $\sqrt{\epsilon} << |\eta \cdot p|$. We note that the expression for $D_{\sigma\rho}(k)$ consider with $D^0_{\sigma\rho}(k)$ except in the small neighborhood ($\sim \alpha\sqrt{\epsilon},\ \alpha >> 1$) of $\eta \cdot k = 0$.Similarly, the expression for $D_{\lambda\kappa}(k+p)$ coincide with $D^0_{\lambda\kappa}(k+p)$ except near $\eta \cdot (k+p) = 0$. For sufficiently small ϵ, t he two regions in the complex k_0-plane are sufficiently separated (separation $\Delta k_0 >> \sqrt{\epsilon}$). Thus,, in the vicinity of $\eta \cdot = 0$, $D_{\lambda\kappa}(k+p) \sim D^0_{\lambda\kappa}(k+p)$ and vice versa. We note that the vertex factor are analytic in k_0. Thus, in the neighborhood $|k_0 - \vec{\eta} \cdot \vec{k}| \sim \alpha\sqrt{\epsilon}(\alpha >> 1)$, the remainder of the integrand (save the factor $D_{\sigma\rho}(k)$ is analytic in k_0 and equals $VVD^0_{\lambda\kappa}(k+p)$ up to $O(\epsilon)$. We can treat the vicinity of the region $\eta \cdot (k+p) = 0$ in a similar manner. We can thus express:

$$\int_{-\infty}^{\infty} dk_0 VVDD. = \int_{C_0} dk_0 VVDD + \int_{C_1} dk_0 VVDD + \int_{C_2} dk_0 VVDD, \quad \text{(A2)}$$

where C_0 is the contour that runs mostly along the real k_0 axis together with two semicircular dips of reads $\alpha\sqrt{\epsilon}$ around the points $\eta \cdot k = 0$ and $\eta \cdot (k+p) = 0$; C_1 and C_2 are compensating closed contours as in [7]. On C, we can replace D by D^0 everywhere in both factors. On C_1, we can replace $D(k+p)$ by $D^0(k+p)$ and on C_2, we can replace $D(k)$ by $D^0(k)$. We can then extract the effective terms fro $\int_{C_1}$ and $\int_{C_2}$ much as in [7]. During the process, we note that $VVD^0(k+p)$ is analytic in k_0 on C_1, etc.] Not surprisingly, we get:

$$\begin{aligned}\int_{-\infty}^{\infty} dk_0 VVDD. &= \int_{C_0} dk_0 VVD^0(k)D^0(k+p) + \int dk_0 VVD^0(k+p)D^{\text{extra}} \\ &+ \int dk_0 VVD^0(k)D^{\text{extra}},\end{aligned} \quad \text{(A3)}$$

where D^{extra} is given in (7). [Here we note the absence of a term involving $VVD^{\text{extra}}D^{\text{extra}}$ for $\eta \cdot p \neq 0$.]

We now discuss the evaluation of the first term on the right hand side side of (A3) and establish a simplified treatment. We note that a typical term involved in this term is of the form:

$$\int_{C_0} dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon]\zeta^\alpha(\zeta+\zeta_0)^\beta},\ 0 \le \alpha, \beta \le 2 \qquad \text{(A4)}$$

where $\zeta = \eta \cdot k$ and $\zeta_0 = \eta \cdot p \neq 0$. Here $P(k,p)$ is a polynomial in k and p. We can always partial fraction $\frac{1}{\zeta^\alpha(\zeta+\zeta_0)^\beta}$, $0 \le \alpha, \beta \le 2$ so that each term has

only single denominator of the the form $\frac{1}{\zeta^\alpha}$ or $\frac{1}{(\zeta+\zeta_0)^\beta}$ $(1 \le \alpha, \beta \le 2)$ [5]

We thus have to evaluate integrals of the form

$$\int_{C_0} dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon]\zeta^\alpha},\ 0 \le \alpha \le 2 \tag{A5}$$

and

$$\int_{C_0} dk_0 \frac{P(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon](\zeta+\zeta_0)^\beta},\ 0 \le \beta \le 2 \tag{A6}$$

in addition to the integrals known for having only local divergences . Note that in each of the above integrals (A5) and (A6), the contour C, now, can be chosen to have only *one* semicircular dip, t he other one having become redundant, because, now, there are poles *either* at $\eta = 0$ *or* at $\zeta + \zeta_0 = 0$. Furthermore, a change of variable $k + p \to k$, casts the latter integral in the form (with C suitably modified)

$$\int_{C_0} dk_0 \frac{P'(k,p)}{(k^2+i\epsilon)[(k+p)^2+i\epsilon]\zeta^\beta},\ 1 \le \beta \le 2 \tag{A7}$$

which is of the same form as (A5).

We thus have to evaluate integrals of the form (A5) with C avoiding the pole at $\zeta = 0$ by encircling it below at a radius $\sim \alpha\sqrt{\epsilon}$.

These are known to be local]2].

Appendix B

In this appendix, we treat the ghost diagrams that we may not neglect for $\lambda \neq 0$. It reads up to overall factors

$$I_G = \int d^4k V_\mu(p,k,-k-p)V_\nu(-p,k+p,-k)D(k)D(k+p) \tag{B1}$$

We can evaluate $D(k)$ along the lines of]7],]8] and establish an effective treatment of (B1). We have (for $\eta \cdot p \neq 0$)

$$I_G = \int d^3k \int_{C_0} dk_0 V_\mu V_\nu D^0(k)D^0(k+p) + \int d^3k \int dk_0 V_\mu V_\nu (D^0(k)D^{\text{extra}}(k+p) + D^{\text{extra}}(k)D^0(k+p)). \tag{B2}$$

[5]We note that in the process of partial fractioning we may generate coefficients of the type $\zeta^{-\gamma}$ $(0 \le \gamma \le 2)$ and these have to be kept track of in the final form.

We recall

$$D^0(k)D^0(k+p) = \frac{1}{\eta\cdot k}\frac{1}{\eta\cdot(k+p)} = \frac{1}{\eta\cdot p}\Big[\frac{1}{\eta\cdot k} - \frac{1}{\eta\cdot(k+p)}\Big]. \tag{B3}$$

Thus, we have to evaluate

$$\int d^3k \int_{C_0} \frac{dk_0}{\eta\cdot k} V_\mu V_\nu, \text{and } \int d^3k \int_{C_0} \frac{dk_0}{\eta\cdot(k+p)} V_\mu V_\nu. \tag{B4}$$

We recall

$$V_\mu(p,k,-k-p) = \eta_\mu + O(\epsilon), \tag{B5}$$

where the $O(\epsilon)$ terms may arise from the $O(\epsilon)$ terms in $\mathrm{S_{eff}}$. In each case, the $O(1)$ terms in the above integrals vanish by the use of results similar to those in (11). We shall assume that the $O(\epsilon)$ terms exist in dimensional regularization and hence can be ignored as $\epsilon \to 0$. Further, calculation along the lines of]7] shows [6]

$$D^{\text{extra}} = \delta\Big(k^0 - \vec{\eta}\cdot\vec{k} - O(\sqrt{\epsilon})\Big)A(k), \tag{B6}$$

the second term in I_G of (B2) becomes:

$$\begin{aligned}
&\int d^3k \int dk_0 V_\mu V_\nu D^0(k)\delta(k^0 + p^0 - \vec{\eta}\cdot(\vec{k}+\vec{p}) + O(\sqrt{\epsilon}))A(k+p) \\
&= \int d^3k \frac{V_\mu V_\nu}{\eta\cdot k} A(k+p)|_{\eta\cdot k = -\eta\cdot p + O(\sqrt{\epsilon})} \\
&= \frac{1}{\eta\cdot p + O(\sqrt{\epsilon})} \int d^3k V_\mu V_\nu A(p+k)|_{\eta\cdot k = -\eta\cdot p + O(\sqrt{\epsilon})}.
\end{aligned} \tag{B7}$$

Again, we note that the $O(1)$ term above vanishes in dimensional regularization. We shall assume that the $O(\sqrt{\epsilon})$ term exists in dimensional regularization and hence can be ignored.

Acknowledgement

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[6] $A(k)$ is dimensionless and in fact turns out to be a constant.

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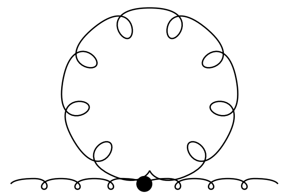

Fig 1(a)

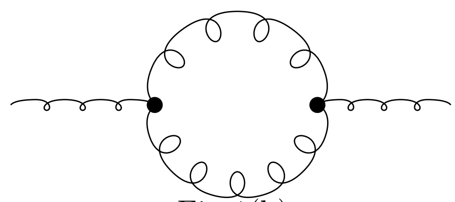

Fig 1(b)

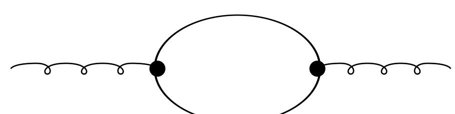

Fig 1(c)